Idea of experiment

Invariance of electrodynamics equations relative to the group of transformations should have close connection with non-invariant properties of transformations for partial differentials of space-time. If the connection has observed consequences, they should appear in moving media electrodynamics: in tasks, where a source, a receiver, boundaries between media and the media move with different velocities.

A fundamental aspect of the question is that moving media electrodynamics equations were tested in some special cases and they weren't tested for 3-dimensional tasks. An applied aspect is an answer on the question: How do readings of an interferometer moving around the Earth depend on its position and orientation? It's cost to be noticed that dependence on a Earth rotation rate was found in an Sagnac-type interferometer.

An analysis of Michelson-Morley-type experiments allow to assert that invariance of the result is provided with very high degree of accuracy. One can made the conclusion after calculating with account: terms of second order smallness β^2, contraction of interferometer length and its elements, alteration of source frequency, and radiation frequency when it reflects from moving elements, variation of reflection angles from moving elements.

The Fizeau's interferometer (fig.1a) is more interesting for analysis. There is no an unique inertial reference frame (IRF), in which all elements rests, therefore, in the case we cannot pass from a rest IRF to a moving IRF. As composition of velocities for the interferometer and the medium should satisfy to relativistic law, it is intrinsic to assume that if non-invariant properties of partial differentials can have observed appearances, they will be found in non-linear terms of a solution of the dispersion equation.

Let us to consider the Fizeau's interferometer in the IRF, in which the interferometer rests, that is $\beta = v/c = 0$, where c is light velocity in vacuum. $\vec{u}$ is water velocity in the interferometer IRF, and $\beta_{2n} = \pm u/c$. The invariants $I_t = k_t = k_0 \sin\vartheta_0 = 0$, $-I_1 = \omega_0(1-\beta) = \omega_0$ corresponds to normal incident beams [1]. In the case parameters $d = \frac{I_t}{I_1} = 0$, $Q = n_2^2$ are contained in the solution of the dispersion equation.

Then a wave vector for a refractive beam is

$$k_{2n} = \frac{\omega_0}{c}\frac{-\beta_{2n}(n_2^2-1)+n_2(1-\beta_{2n}^2)}{1-n_2^2\beta_{2n}^2}. \qquad (1)$$

Difference between beam passages will depend on time of light propagation in opposite directions:

$$\Delta_0 = \frac{c}{\lambda}(t_2-t_1) = \frac{lc}{\lambda\omega_0}(k_{2n,2}-k_{2n,1}) = \frac{4l}{\lambda}\frac{\beta_{2n}(n_2^2-1)}{1-n_2^2\beta_{2n}^2}. \qquad (2)$$

For parameters of the Fizeau's experiment [2] $l = 1{,}4875$ m, $u = 7{,}059$ m/s, $\lambda = 0{,}526$ mkm, $n_2 \approx 1{,}33$ we have got $\Delta_0 = 0{,}170$. A shift $\Delta = 0{,}23$ was observed in the Fizeau's experiment, the value is explained with the fact that water velocity along an axis of the tube was more than an average value for u which was used in calculations. A second order term $n_2^2\beta_{2n}^2 = 1{,}7\times10^{-15}$ is very small and it doesn't influence on results.

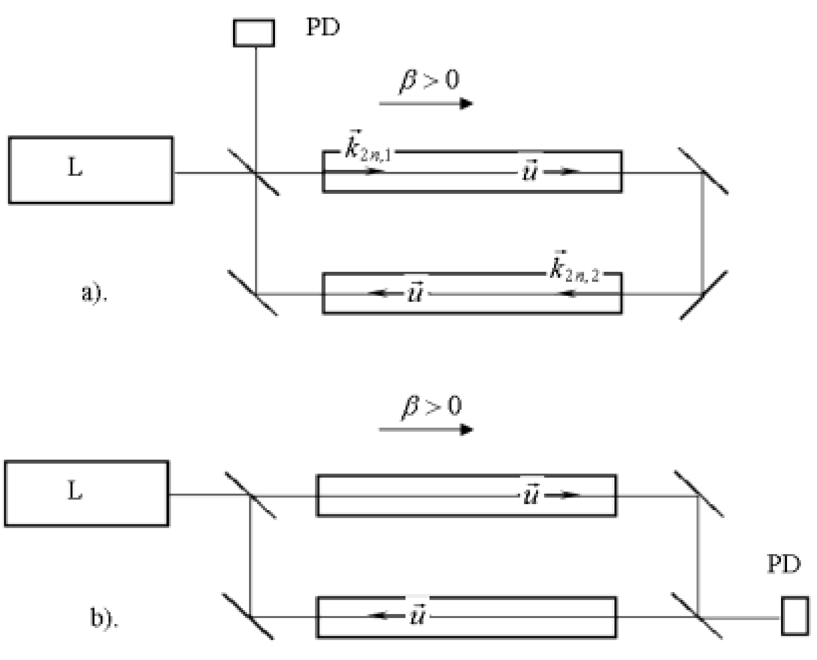

Fig. 1. Schemes of interferometers with two passages (a) and single passage (b), in the interferometers light from a laser L propagates in a moving medium with velocity $\vec{u}$. A photodetector PD register interference fringes (IF). Velocity $\vec{u}$ is given in an observer IRF. The interferometer moves with velocity $\vec{v}$ to the right

($\beta = v/c > 0$) or to the left ($\beta < 0$) relative to the observer IRF.

Let us consider the interferometer moving with velocity v relative to the IRF. First of all e will consider the case when light beams pass a tube one time (fig.1.b). Then, $-I_1 = \omega_1(1-\beta)$, here ω_1 is a source frequency in a n observer IRF. The expression (1) will have a view

$$k_{2n} = \frac{\omega_1}{c}(1-\beta)\frac{\beta + \left(n_2^2 - 1\right)\dfrac{\beta - \beta_{2n}}{1-\beta_{2n}^2} + n_2}{1-\beta^2 - \left(n_2^2-1\right)\dfrac{\left(\beta-\beta_{2n}\right)^2}{1-\beta_{2n}^2}}. \qquad (3)$$

Here u is velocity in IRF where the interferometer moves. The velocity in an interferometer IRF is u', in the case

$$\beta_{2n} = \frac{\beta + \beta'_{2n}}{1+\beta\beta'_{2n}}. \qquad (4)$$

By substituting (4) in (3), we will get

$$k_{2n,1} = \frac{\omega_1}{c}\frac{\beta + \beta'_{2n} - n_2^2\beta'_{2n}(1+\beta\beta'_{2n}) + n_2(1-\beta'^2_{2n})}{(1+\beta)\left(1-n_2^2\beta_{2n}^2\right)}, \qquad (5)$$

A sign before β_{2n} changes in the expression. Path difference will be calculated

$$\Delta = \frac{2l}{\lambda_1}\frac{\beta'_{2n}(n_2^2-1)}{(1+\beta)\left(1-n_2^2\beta'^2_{2n}\right)}. \qquad (6)$$

Due to Doppler's effect a wave length is equal to $\lambda_1 = \lambda_0\sqrt{\dfrac{1-\beta}{1+\beta}}$. Taking into account a kinematical shift of the interferometer, a path in a medium increases $l = l_1/(1-\beta)$, and also there is the contraction effect $l_1 = l_0\sqrt{1-\beta^2}$. A resulting shift of IF is equal to

$$\Delta = \frac{2l_0}{\lambda_0}\frac{\beta'_{2n}(n_2^2-1)}{(1-\beta)\left(1-n_2^2\beta'^2_{2n}\right)}. \qquad (7)$$

Difference in interferometer readings when $\beta = 0$ and $\beta \neq 0$ will be equal to

$$\Delta - \Delta_0/2 \approx \beta\Delta_0. \qquad (8)$$

Thus, maximal variations for the IF shift in the interferometer moving relative to the Sun with $\beta \simeq 10^{-4}$ and with different orientations of the interferometer to velocity vector would have order of a value $\delta\Delta = \pm\beta\Delta_0 = \pm 1,7\times 10^{-5}$ (of fringe).

Let us consider a complete scheme of the Fizeau's interferometer and estimate an IF shift with account dispersion in a material. In the case we have

$$\Delta = \frac{l}{\lambda k_0}\left(\frac{1}{1-\beta}k_{2n,1} + \frac{1}{1+\beta}k_{2n,2} - \frac{1}{1-\beta}k_{1n,1} - \frac{1}{1+\beta}k_{1n,2}\right) \qquad (9)$$

In limit $\beta'^2_{2n} \to 0$ and $\beta\beta'_{2n} \to 0$ we have

$$k_{2n,1} = k_{01}\frac{n_{2,1} + \beta + \beta'_{2n}\left(n_2^2 - 1\right)}{1+\beta}, \qquad (10a)$$

$$k_{2n,2}=k_{02}\frac{n_{2,2}-\beta+\beta'_{2n}\left(n_2^2-1\right)}{1-\beta}, \quad (10b)$$

$$k_{1n,1}=k_{01}\frac{n_{1,1}+\beta-\beta'_{2n}\left(n_2^2-1\right)}{1+\beta}, \quad (10c)$$

$$k_{1n,2}=k_{01}\frac{n_{1,2}-\beta-\beta'_{2n}\left(n_2^2-1\right)}{1-\beta}. \quad (10d)$$

Here are $k_{01}=k_0\sqrt{\frac{1+\beta}{1-\beta}}\approx k_0\left(1+\beta\right)$, $\qquad k_{02}=k_0\sqrt{\frac{1-\beta}{1+\beta}}\approx k_0\left(1-\beta\right)$. (11)

For the case when $\beta>>\beta_{2n}$ refractive indexes are $n_{2,2}\approx n_{1,2}$, $n_{2,1}\approx n_{1,1}$. Then substitution to (9) gives

$$\Delta=\frac{2l\beta'_{2n}}{\lambda}\frac{\left(n_{1,1}^2-1\right)\left(1+\beta\right)+\left(n_{1,2}^2-1\right)\left(1-\beta\right)}{1-\beta^2}. \quad (12)$$

In non-dispersion approximation when $n_{1,1}=n_{1,2}=n$ we will get

$$\Delta=\frac{4l\beta'_{2n}}{\lambda}\frac{\left(n^2-1\right)}{1-\beta^2}. \quad (13)$$

As the expression was received for limits $\beta'^2_{2n}\to 0$ and $\beta\beta'_{2n}\to 0$, the term β^2 can be taken into account, hence, it gives slight contribution to interference fringe shift and it is equal to $\beta^2\Delta_0$.

Influence of dispersion may be estimated in the first approximation in the following way

$$n_{2,2}\approx n_{1,2}\cong n+\delta,\ n_{2,1}\approx n_{1,1}\cong n-\delta,\ \ \delta=\Delta n-\delta n, \quad (14)$$

where Δn is variation of refractive index n due to motion of a boundary between two media, δn is variation of refractive index n due to length difference of waves which are incident onto the boundary.

By taking into account the dispersion from the expression (12) we have

$$\Delta=\frac{4l\beta'_{2n}}{\lambda}\frac{\left(n^2+\delta^2-2\beta n\delta-1\right)}{1-\beta^2}. \quad (15)$$

As the variations Δn and δn have different signs, we can neglect δ^2, more over $\delta^2<<n^2$ and the expression (15) can be reduced to the classical result (13).

Let us write down exact expressions for wave vectors and frequencies, which contain β'^2_{2n} and $\beta\beta'_{2n}$ to estimate influence of dispersion more precisely. Also, we will use an experimental tested dependence for a refractive index of optical glass on wave length of radiation.

In the case the expressions (10) will take a view

$$k_{2n,1}=k_{01}\frac{\beta-\beta'_{2n}+n_{2,1}^2\beta'_{2n}\left(1-\beta\beta'_{2n}\right)+n_{2,1}\left(1-\beta'^2_{2n}\right)}{\left(1+\beta\right)\left(1-n_{2,1}^2\beta'^2_{2n}\right)}, \quad (16a)$$

$$k_{2n,2}=k_{02}\frac{-\beta-\beta'_{2n}+n_{2,1}^2\beta'_{2n}\left(1+\beta\beta'_{2n}\right)+n_{2,1}\left(1-\beta'^2_{2n}\right)}{\left(1-\beta\right)\left(1-n_{2,1}^2\beta'^2_{2n}\right)}, \quad (16b)$$

$$k_{1n,1}=k_{01}\frac{\beta+\beta'_{2n}-n_{1,1}^2\beta'_{2n}\left(1+\beta\beta'_{2n}\right)+n_{1,1}\left(1-\beta'^2_{2n}\right)}{\left(1+\beta\right)\left(1-n_{1,1}^2\beta'^2_{2n}\right)}, \quad (16c)$$

$$k_{1n,2} = k_{02} \frac{-\beta + \beta'_{2n} - n_{1,2}^{2} \beta'_{2n} \left(1 - \beta \beta'_{2n}\right) + n_{1,1} \left(1 - \beta'^{2}_{2n}\right)}{\left(1-\beta\right)\left(1 - n_{1,2}^{2} \beta'^{2}_{2n}\right)}. \quad (16d)$$

Wave numbers are defined with a method of successive approximations. First of all an refractive index, which was measured in a IRF where a medium rests, is substituted in the expression (16). Moreover frequency of an incident radiation is defined. Then corresponding wave lengths are calculated in a moving medium.

$$\lambda_{1,1} = \frac{2\pi c}{k_{1n,1} v + \omega_1 \left(1-\beta\right)}, \quad \lambda_{1,2} = \frac{2\pi c}{-k_{1n,2} v + \omega_1 \sqrt{\frac{1-\beta}{1+\beta}}}, \quad (17a)$$

$$\lambda_{2,1} = \frac{2\pi c}{k_{2n,1} v + \omega_1 \left(1-\beta\right)}, \quad \lambda_{2,2} = \frac{2\pi c}{-k_{2n,2} v + \omega_1 \sqrt{\frac{1-\beta}{1+\beta}}}. \quad (17b)$$

A refractive index is found for each wave length, for example, for $n_{1,1}$ the expression will correspond to:

$$n_{1,1}^{2} = A_1 + A_2 \lambda_{1,1}^{2} + A_3 \lambda_{1,1}^{-2} + A_4 \lambda_{1,1}^{-4} + A_5 \lambda_{1,1}^{-6} + A_6 \lambda_{1,1}^{-8}. \quad (18)$$

Coefficients A_i are selected with respect to experimental results. The indexes $n_{1,2}$, $n_{2,1}$, $n_{2,2}$ are analogically calculated. Then the indexes are substituted into (16) for the second time and wave numbers are calculated. When it is needed to increase accuracy of results t.e procedure can be repeated. The results of numerical experiments are presented in a table 1. Common parameters for all schemes were $l = 1,4875$ì , $u = 7,059$ì/ñ , $\beta = (V_z + V_s)/c$, here V_z and V_s are a daily velocity of the Earth, and an orbital velocity of the Sun. An refractive index for water $n_2 = 1,3314$ was taken in the experiment. Thus, water dispersion wasn't taken into account.

When we used the glass LK5, a refractive index was calculated with the formula (18) for each beam and passage in dependence on a motion direction and a frequency of incident radiation, respectively. It has average value $n_2 = 1,476615$. Approximation without dispersion meant that refraction onto a moving boundary between two media was calculated for a refractive index and. In a real case after refraction onto a moving boundary between two media a frequency of incident light changed that leads to recalculate a refractive index for a moving medium. The results of calculations with dispersion on a boundary between two media are collected in the third and sixth rows of the table. Values Δ and Δ' are presented as absolute those.

Table 1.

Type of interferometer, Its parameters	**Shift of interference fringes**		$\Delta' - \Delta$	$\Delta' + \Delta,\ 10^{-1}$
	$\beta > 0$	$\beta < 0$		
	$\Delta,\ 10^{-1}$	$\Delta',\ 10^{-1}$		
1. One-passage, $\lambda = 0,526$ mkm, water without dispersion	1,0283246	1,0288774	5,53$\times 10^{-5}$	2,057202
2. One-passage, $\lambda = 0,6328$ mkm, LK5, approximation without dispersion	1,3054644	1,3062296	7,65$\times 10^{-5}$	2,611694
3. One-passage, $\lambda = 0,6328$ mkm, LK5 with dispersion	1,3056019	1,3060920	4,90$\times 10^{-5}$	2,611694
4. Two-passages, $\lambda = 0,526$ mkm, water	1,7094844	1,7094844	0	3,4189688

without dispersion				
5. Two-passages, $\lambda = 0{,}6328$ mkm, LK5, approximation without dispersion	2,6117071	2,6116809	-2,62 $\times 10^{-6}$	5,2233879
6. Two-passages, $\lambda = 0{,}6328$ mkm, LK5 with dispersion	2,6117186	2,6116694	-4,92 $\times 10^{-6}$	5,2233879

First of all it is necessary to notice that the sum $\Delta' + \Delta$ is equal to the value given in the corresponding column for all schemes with $\beta = 0$. Therefore, resulting shift of IF doesn't depend on the fact an interferometer moves or doesn't move when a direction of motion is changed. Moreover, the difference $\Delta' - \Delta$ is equal to zero in the case.

It can be noticed from the given values Δ and Δ' in the table that the values Δ and Δ' have some difference for different signs $\beta > 0$ or $\beta < 0$.

In the first scheme the difference $\Delta' - \Delta$ is equal to $2\delta\Delta \cong 2\beta\Delta_0$, which was obtained from the expression (8). The magnitude 5,53 $\times 10^{-5}$ is less than an error in the Fizeau's experiment in three orders. The result was received without account dispersion in moving water. As it is difficult to take into account dispersion in water, we used light glass LK5, for which dispersion coefficients were experimentally defined.

In the second row of the table results are given in approximation without dispersion. The difference $\Delta' - \Delta$ increased due to a coefficient $(n^2 - 1)$ was larger for the glass.

In the third row of the table results are given with dispersion. The dispersion in material of moving glass decreased the difference $\Delta' - \Delta$ on 36%. Estimation of dispersion influence was carried out for stationary glass and dispersion coefficients provided calculation error $\pm 1 \times 10^{-5}$ which was calculated with the expression (18).

In two passages schemes the difference $\Delta' - \Delta$ decreased due to compensation of light dragging effects in a moving medium for opposite direction of motion. Really, the difference is equal to zero in the fourth row, the value in the fifth and sixth rows is considerable less than in the one-passage scheme with dispersion. In whole, it can be concluded that variations of Δ are slight and the maximal value $\Delta' - \Delta$ is equal to 4,9 $\times 10^{-5}$. We can notice that that variations of $\Delta' - \Delta$ depend on refractive index n_2, length l and velocity $\vec{u}$ and $\vec{\beta}$. Carrying out a similar experiment can allow to find out is there a dependence $\Delta' - \Delta$ on spatial orientation of an interferometer. If a result is zero, we can define maximal limit for $\vec{\beta}$ in the case.

The given schemes of an interferometer are not optimal from a view point of experiment. The one-passage scheme is not stable to perturbing factors; the two-passage scheme has low sensibility. Besides, there are interferometers which have several orders higher measurement accuracy, for an example, interferometers for gravitational wave detection [3]. But distinctive peculiarity of interferometers, which are interesting for us, is availability of a moving medium. The medium will bring in vibrations in an interferometer, hence, as measured value of IF shift is not connected with motion of an element, we can use a compensation scheme when motion of any element leads to the same influence on each interfering beam.

A scheme with a rotating disc, close to given that [4], can be alternative. As light propagates in a rotating disc with 3-dimational presentation of velocity, we can use corresponding integral equations [5] for precise description. In such schemes we can observe violation of the Snell's law, which can considerably influence on results, especially with account dispersion. Carrying out the experiment could provide testing electrodynamics equations with 3-dimational presentation of velocity law.

This calculation was published in [6].

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