Gauge of an interferometer

For the gauge of the interferometer we need to find link between shift of interference fringes and time signal from photodetector along which fringes cyclically move.

Let us consider interference fringes of equal thickness. Let us the interferometer is adjusted that the average position of the photodetector in interference pattern plane is characterized with the coordinate x_n (fig.1). Fringes move relative the position on left and right with the amplitude A_0 and the frequency $\omega = 2\pi\nu$:

$$x(t) = x_n + A_0 \cos\omega t\,. \qquad (1)$$

Intensity distribution in the plane *PD2* has a view (Fig.2a)

$$I(x) = \frac{1}{2}\left[I_0 + I_T + (I_0 - I_T)\cos\left(\Omega x + \delta\right)\right], \qquad (2)$$

where I_0, I_T are intensities of interference fringes in the maximum and the minimum, $\Omega = 2\pi/x_p$ is spatial frequency for fringes, x_p is width of a fringe, $\delta = \Omega\,\Delta x = \frac{2\pi\,\Delta x}{x_p}$ is the relative shift of the interference pattern due to light dragging effect expressed in radians, Δx is absolute shift of an interference pattern.

Interference pattern shift due to the longitudinal Fizeau's effect is calculated [2]

$$\Delta_0 = \frac{\Delta x}{x_p} = \frac{2l}{\lambda}\frac{\beta_{2n}(n^2-1)}{1-n^2\beta_{2n}^2}\,. \qquad (3)$$

where $\beta_{2n} = V_{2n}/c = 0$, V_{2n} is the projection of medium velocity on a wave vector, c is light velocity in vacuum. Interference pattern shift with accounting the transversal dragging effect

$$\Delta_\Sigma^\pm = (1 \pm \rho)\Delta_0\,. \qquad (4)$$

Here the parameter is

$$\rho\left(n_2, \vartheta_0\right) = \frac{n_2 - 1}{n_2} tg\vartheta_2 = \frac{n_2 - 1}{n_2}\frac{\sin^2\vartheta_0}{\sqrt{n_2^2 - \sin^2\vartheta_0}} \qquad (5)$$

Sign in the formula (4) is defined with the scheme of the interferometer. For the interferometer in the figure 1 the longitudinal and transversal dragging effects have different directions, so we need to select the sign «-» and obtain the result $\Delta_\Sigma^- = 0{,}017\ldots0{,}024$ for rates of OD $\nu = 250\ldots350\,Hz$.

Estimation for fringes shift variations in the interferometer when it rotates in space with $\beta \cong 2{,}3\times10^{-3}$ gives the magnitude order $d\Delta = 2\beta\Delta_\Sigma^- = (0{,}78\ldots1{,}10)\times10^{-4}$ (of a fringe) without accounting influence of dispersion in a moving medium [3]. Therefore, the needed level of sensibility is $d\Delta \approx 3\times10^{-5}$.

The time dependence of a signal from compact inertialess photodetector has a view (Fig.1b)

$$I(t) = \frac{1}{2}\left[I_0 + I_T + (I_0 - I_T)\cos\left(\Omega\left(x_n + A_0\cos\omega t\right) + \delta\right)\right]. \qquad (6)$$

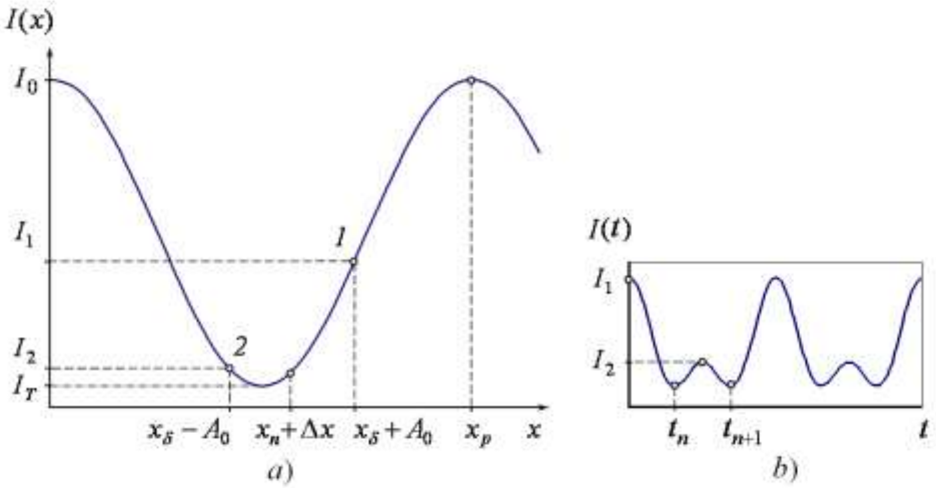

Fig. 1. Dependence of interference fringe intensity on a coordinate $I(x)$ (a) and dependence of intensity on photofedector on time $I(t)$ (b).

The useful signal is time interval between neighboring minimums $\Delta t = t_{n+1} - t_n$, its magnitude depends on velocity of OD rotation and also on Δx.

For coordinates of dark fringes (fig. 1b) from the expression $\dfrac{dI}{dt} = 0$ we have

$$t_n = \frac{1}{\omega}\arccos\frac{x_n\Omega + \delta - \pi n}{\Omega A_0}, \qquad t_{n+1} = \frac{1}{\omega}\arccos\frac{x_n\Omega + \delta - \pi(n+1)}{\Omega A_0}. \tag{7}$$

The interval Δt depends on the period of OD rotation, so we find it value normed to the period T

$$\frac{\Delta t}{T} = \frac{1}{2\pi}\left\{\arccos\frac{x_n\Omega + \delta - \pi(n+1)}{\Omega A_0} - \arccos\frac{x_n\Omega + \delta - \pi n)}{\Omega A_0}\right\}. \tag{8}$$

Let us present the derivation on δ for the expression (8):

$$\frac{d}{d\delta}\left(\frac{\Delta t}{T}\right) = \frac{1}{2\pi}\frac{\sqrt{\left(A_0^2 - \left(x_n + \Delta x - x_p\, n/2\right)^2\right)} - \sqrt{\left(A_0^2 - \left(x_n + \Delta x - x_p\,(n+1)/2\right)^2\right)}}{\sqrt{\left(A_0^2 - \left(x_n + \Delta x - x_p\,(n+1)/2\right)^2\right)\left(A_0^2 - \left(x_n + \Delta x - x_p\, n/2\right)^2\right)}} \tag{9}$$

The given expression defines sensibility of the interferometer to shift of fringes. Really, the variation of time signal $d\Delta t/T$ will be maximal with shift of fringes $d\delta$, if any condition from these bellow is satisfied

$$A_0 = x_n + \Delta x - x_p\, n/2, \qquad A_0 = x_n + \Delta x - x_p\left(n+1\right)/2. \tag{10}$$

Let us $n = 1$, and introduce designations $x_n + \Delta x = x_\delta$, $\tilde{A}_0 = \dfrac{A_0}{x_p}$, $\tilde{x}_\delta = \dfrac{x_\delta}{x_p}$.

Then the expression (6) can be rewritten

$$\frac{d}{d\Delta}\left(\frac{\Delta t}{T}\right) = \frac{\sqrt{4\tilde{A}_0^2 - (2\tilde{x}_\delta - 1)^2} - 2\sqrt{\tilde{A}_0^2 - \tilde{x}_\delta^2}}{2\sqrt{\left(4\tilde{A}_0^2 - (2\tilde{x}_\delta - 1)^2\right)\left(\tilde{A}_0^2 - \tilde{x}_\delta^2\right)}}. \tag{11}$$

Let us $4\tilde{A}_0^2 - \left(2\tilde{x}_\delta - 1\right)^2 = \tilde{d}^2$, where $\left|\tilde{d}\right|$ is small,

$$\frac{d}{d\Delta}\left(\frac{\Delta t}{T}\right) = \frac{\sqrt{1 - 4\tilde{x}_\delta + \tilde{d}^2} - \tilde{d}}{\tilde{d}\sqrt{1 - 4\tilde{x}_\delta + \tilde{d}^2}}. \tag{12}$$

Let us $1-4\tilde{x}_\delta >> \tilde{d}^2$, that is not easy to fulfill, then the shift of fringes can be estimated as

$$d\Delta = d\left(\frac{\Delta t}{T}\right)\tilde{d}\,. \qquad (13)$$

Let us make assumption that minimal detected signal of interference fringes shift is on the level of noise for measuring the time interval Δt. Let us the magnitude is known from an experiment and equal to $d\left(\frac{\Delta t}{T}\right)=10^{-4}$, and interferometer adjustment corresponds to $\tilde{d}=10^{-1}$, supposing that signal-noise relation is equal to $S/N=3$, we can detect the signal $d\Delta = 3\times 10^{-5}$ (of a fringe).

In the experiment we observed noise $d\left(\frac{\Delta t}{T}\right)=(1...7)\times 10^{-4}$. Therefore, in the best experiment when $d\left(\frac{\Delta t}{T}\right)=10^{-4}$ and adjustment $\tilde{d}=10^{-1}$ we can detect the shift of fringes $d\Delta = 3\times 10^{-5}$ which connected with spatial anisotropy with $\beta \cong 2{,}3\times 10^{-3}$.

This gauge of an interferometer method was published in [3].

References

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[3]. Gladyshev V.O., Sharandin E.A., Gladysheva T.M., Tiunov P.S., Leontyev A.D., Podguzov G.V. Interference optical experiments for finding space anisotropy // Physical Interpretation of Relativity Theory : Proceedings of XV International Meeting. – Moscow: BMSTU, 2009. p. 215-223.