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"Our conceptual universe is merely the simplest logical construct into which we can gather all known, perceived phenomena..."

Karl Pearson

ON INFLUENCE OF SPACE-TIME CURVATURE FLUCTUATIONS ON LIGHT PROPAGATION IN UNIVERSE

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1. INTRODUCTION

The relativity predicts gravitational waves from neutron stars, black holes and other astrophysical sources. Experimental detecting gravitational waves (GW) are carried out since 1960th, hence there are still not satisfied evidence of its detection. Thus the requirement of creating new experimental tests for verification our idea on space-time continuum arises.

If the gravitational waves exist, fluctuations of space curvature must appear in the space-time. The fluctuations are the consequence of composition of the large quantity GW with different frequencies and amplitudes.

Let consider the process of propagating light in the space-time with fluctuating metric tensor. We will propose that metric tensor $g_{ik}\left(x^0,x^1,x^2,x^3\right)$ presents random coordinate function $x^l = \left(x^0,x^1,x^2,x^3\right)$. It is to define the character of passing the researched physical process in case, when metric tensor $g_{ik}\left(x^l\right)$ alters in random manner in course of time and with shift in the space.

2. DESCRIPTION OF LIGHT PROPAGATION IN SPACE-TIME WITH FLUCTUATING METRIC

By creating the equations, which describe physical processes in space-time with fluctuating metric tensor, we can use the method, presented in the work [2]. The equations for electromagnetic field in the curved space-time allow analyze the peculiarities of electromagnetic wave propagation in the space-time with fluctuating metric in particular.

Furthermore, let consider the case of light propagation in the geometric optics approximation. This case can take place under the condition, when the light wavelength λ_0 is much less than the characteristic dimensions δL of metric tensor fluctuations: $\lambda_0 << \delta L$. The equation describing the light propagation in the curved space-time for the case [2]:

$$\frac{dk^i}{d\chi} + \Gamma^i_{kl} k^k k^l = 0, \tag{1}$$

where: k^i - four-dimensional wave vector, χ - the parameter, altering along a beam, Γ^i_{kl} - Christoffer's symbols, which are expressed via metric tensor in the next way:

$$\Gamma^i_{kl} = \frac{1}{2} g^{im}\left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right). \tag{2}$$

In the formulae (1), (2) and after it is supposed to sum over all repeating indexes. The metric tensor g_{ik} is the symmetric one: $g_{ik} = g_{ki}$.

The tensor g^{lk} is depended on the tensor g_{il} with the relation:

$$g_{il} g^{lk} = \delta^k_i, \tag{3}$$

where δ_i^k -unit tensor or Kronecker's δ - function.

Furthermore, let consider the case, when metric tensor g_{ik} is little different from Galilei's metric $g_{ik}^{(0)}$:

$$g_{ik} = g_{ik}^{(0)} + h_{ik} , \tag{4}$$

where: $|h_{ik}(x^l)| << 1$, $g_{00}^{(0)} = 1$, $g_{11}^{(0)} = g_{22}^{(0)} = g_{33}^{(0)} = -1$; $g_{ik}^{(0)} = 0$ with $i \neq k$. Christoffer's symbols in the first approximation can be:

$$\Gamma^i_{kl} = \frac{1}{2} g^{im}_{(0)} \left(\frac{\partial h_{mk}}{\partial x^l} + \frac{\partial h_{ml}}{\partial x^k} - \frac{\partial h_{kl}}{\partial x^m} \right) , \tag{5}$$

where $g^{im}_{(0)} = g_{im}^{(0)}$.

Following the work [3] the solution of equation (1) will be found in the form a row:

$$k^i = k^i_{(0)} + k^i_{(1)} + ..., \tag{6}$$

where $|k^i_{(1)}| << |k^i_{(0)}|$.

So, the equation (1) will be written in the zeroth approximation as:

$$\frac{dk^i_{(0)}}{d\chi} = 0 . \tag{7}$$

Therefore, $k^i_{(0)} = const$.

In the first approximation integrating the equation (1) gives

$$k^i(\chi) = k^i_{(0)} + k^i_{(1)}(0) - k^k_{(0)} k^l_{(0)} \int_0^\chi \Gamma^i_{kl} d\chi . \tag{8}$$

The expression for the four-dimensional vector $k^i_{(1)}(0)$ may be obtained from the condition [2]:

$$g_{ik} k^i k^k = 0 \tag{9}$$

and in the first approximation it is as follows:

$$k^i_{(1)} = -\frac{1}{2} g^{im}_{(0)} h_{ml}(0) k^l_{(0)} . \tag{10}$$

Then the solution (8) gets the final form:

$$k^i(\chi) = k^i_{(0)} - \frac{1}{2} g^{im}_{(0)} h_{ml}(0) k^i_{(0)} - k^k_{(0)} k^l_{(0)} \int_0^\chi \Gamma^i_{kl} d\chi . \tag{11}$$

As the vector k^i may be presented like [2]

$$k^i = \frac{dx^i}{d\chi} , \tag{12}$$

so, in the first approximation we have:

$$x^i(\chi) = k^i_{(0)} \chi + x^i(0) . \tag{13}$$

Let consider light beam propagation in a direction of the axis x^1. Then we may consider, that the vector components $k^i_{(0)}$ have the next values: $k^0_{(0)} = k^1_{(0)} = k_0$, $k^2_{(0)} = k^3_{(0)} = 0$, where k_0 - the wave number. Accounting the distance l, which was passed by light, $l = x^1(\chi) - x^1(0)$, using the formula (13) with $i = 1$, we get the expression for the parameter χ:

$$\chi = \frac{l}{k_0}. \tag{14}$$

The taken propositions allow obtain the next expressions, using (11):

$$\frac{k^{\alpha}(l)}{k_0} = 1 - \frac{1}{2}\left(1^{\alpha}\left(h_{0\alpha}(0) + h_{1\alpha}(0)\right)\right) - \int_0^l \left(\Gamma^{\alpha}_{00} + 2\Gamma^{\alpha}_{01} + \Gamma^{\alpha}_{11}\right) dl, \quad \alpha = 0,1 , \tag{15}$$

$$\frac{k^{\beta}(l)}{k_0} = \frac{1}{2}\left(h_{0\beta}(0) + h_{1\beta}(0)\right) - \int_0^l \left(\Gamma^{\beta}_{00} + 2\Gamma^{\beta}_{01} + \Gamma^{\beta}_{11}\right) dl, \quad \beta = 2,3 , \tag{16}$$

where the symmetry of Christoffel's symbols $\Gamma^i_{kl} = \Gamma^i_{lk}$ was taken into account.

By substituting the expression (5) of Christoffel's symbols into the formulae (15) and (16), we can write the expressions for relative fluctuations of frequency
$\delta\omega$ and the wave vector δk^{α} in the observation point:

$$\frac{\delta\omega}{\omega_0} = \frac{\delta k^0}{k_0} = -\frac{1}{2}\left(h_{00}(0) + h_{01}(0)\right) + \int_0^l \left(\frac{1}{2}\frac{\partial\left(h_{11} - h_{00}\right)}{\partial x^0} - \frac{\partial\left(h_{00} + h_{01}\right)}{\partial x^1}\right) dl, \tag{17}$$

$$\frac{\delta k^{\alpha}}{k_0} = \frac{1}{2}\left(h_{0\alpha}(0) + h_{1\alpha}(0)\right) +$$
$$+ \int_0^l \left(\frac{\partial\left(h_{0\alpha} + h_{1\alpha}\right)}{\partial x^0} + \frac{\partial\left(h_{0\alpha} + h_{1\alpha}\right)}{\partial x^1} - \frac{1}{2}\frac{\partial\left(h_{00} + 2h_{01} + h_{11}\right)}{\partial x^{\alpha}}\right) dl, \alpha = \overline{1,3}. \tag{18}$$

Taking into account that the formulae (13) and (14) allow to write $dx^1 = dl$, we integrate the second term in the sum of the integral of expression (17). Than

$$\frac{\delta\omega}{\omega_0} = \frac{1}{2}\left(h_{00}(0) + h_{01}(0)\right) - h_{00}(l) - h_{01}(l) + \frac{1}{2}\int_0^l \frac{\partial\left(h_{11} - h_{00}\right)}{\partial x^0} dl . \tag{19}$$

Let present the tensor h_{ik} in the form

$$h_{ik} = h\xi_{ik} , \quad i,k = 0,1 , \tag{20}$$

where: h - the fluctuation amplitude of the tensor h_{ik}, $\xi_{ik} = \xi_{ik}(x^0, x^1)$ - the random field with the unit amplitude and the correlation radius $\delta L < \lambda_0$. In this case all summands, standing ahead of the integral in the expression (19), will magnitude of the order h.

Let estimate the especial values of the integral in the expression (19). As limiting values of correlation dimensions $\delta L \approx \lambda_0 = 1/k_0$ and the correlation time constant $\delta\tau \approx 1/\omega_0$ bound applying the geometric optics approximation, so the estimation of frequency fluctuations, which is by the integral in the formula (19), takes a look

$$\sigma^2_{\omega} \approx h^2 l k_0 \tag{21}$$

or

$$h \approx \frac{\sigma_{\omega}}{\sqrt{l k_0}} . \tag{22}$$

When $l >> 1/k_0$, contribution of the integral to the light frequency fluctuations becomes to be determining.

Let do estimations of line broadening, when light passes through space with fluctuating metric. If the value l is about equal to the characteristic dimension of the Universe: $l \approx 10^{25}...10^{26}$ m (1…10 billions of light years), $k_0 = 10^7$ rad /m, and the minimum registrated line broadening

$\sigma_\omega \approx 10^{-6}...10^{-8}$, so the fluctuation amplitude h, which is calculated by the formula (22), will have the magnitude: $h \approx 10^{-22}...10^{-24}$.

In such a way, by propagating light the distance, correlated with Universe dimensions, in case, when the fluctuations of metric has the amplitude $h \approx 10^{-22}...10^{-24}$ and the frequency, limited light wave frequency at the top, it should be observed the boardening of spectral line profiles. Registration of irreversible process of spectral line boardening may be one of evidence of existence of gravitational waves in the nature.

If description is carried out in the synchronous frame [2] as it is usually used for calculating the effect of planar gravitational waves in the frameworks of linear theory [3], so from the condition of Lorentz's calibration it follows:

$$h_{00} = h_{0i} = 0 \ . \qquad (23)$$

Than the expression (19) for relative fluctuations of light frequency gets the view

$$\frac{\delta\omega}{\omega_0} = \frac{1}{2}\int_0^l \frac{\partial h_{11}}{\partial x^0} dl \ . \qquad (24)$$

Considering that the gravitational waves, fulfilling space, have the boundary frequency ω_g and the wave number k_g according to the frequency, the dispersion of light frequency fluctuations may be estimated, using the formula

$$\sigma_\omega \approx h\sqrt{lk_g} \approx h\sqrt{\frac{l\omega_g}{c}} \ , \qquad (25)$$

where h - the gravitational wave characteristic amplitude.

3. CONCLUSION

Analysis of the expression (25) allows to conclude that the potentially registrated fluctuations of light frequency can arise in the case, when there are reasonably high-frequency gravitational waves, in particular the relict gravitational radiation, linked with the processes close to the cosmological peculiarity. The resolution, which is needed to registrate line profile boardening, is in the level achieved in the research laboratories, of spectral resolution $10^6...10^8$ and below the theoretical resolution in several orders [4].

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